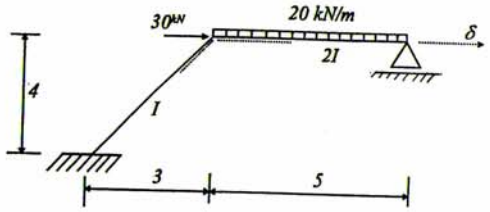
Örnek: şekildeki hiperstatik sistemin

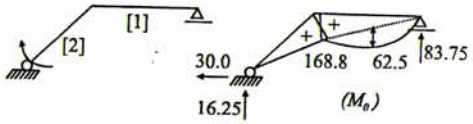
a) M diyagramını çizimi

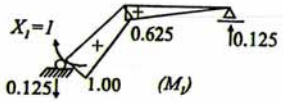
b) δ_x yatay yerdeğiştirmesinin hesabı;



Not 1: $EI=100000\ kNm^2$

2: Uzama ve kayma şekildeğiştirmeleri terkedilecektir.





$$EI_c\delta_{11}=\frac{1}{3}5(1^2+0.625+0.625^2)[2]$$

$$\ldots+\frac{1}{3}5*0.625^2[1]=7.37$$

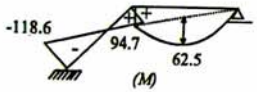
$$EI_c\delta_{10}=\frac{1}{6}5*168.8(1+2*0.625)[2]+\ldots$$

$$\frac{1}{3}5*168.8*0.625[1]+\frac{1}{3}5*62.5*0.625[1]$$

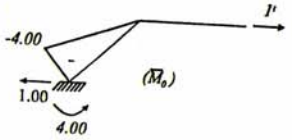
$$=873.9$$

$7.37X_1+873.9=0 \quad X_1=-118.6$

$M=M_0-118.6M_1$



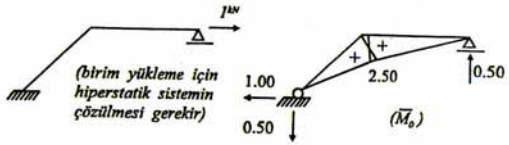
b3) Kısaltma teoreminin uygulanması ve $(\bar{M})_0$ diyagramının herhangi bir izostatik sistemden alınması hali



$$EI_c\delta_x=\frac{1}{6}5*4(2*118.6-94.7)[2]$$

$$EI_c\delta_x=950.0\rightarrow\delta_x=0.0048\ m$$

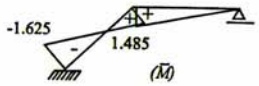
b1) $\bar{M}$ diyagramının hiperstatik sistemden alınması hali (kısaltma teoreminden yararlanılmaması hali)



$$EI_c\delta_{10}=\frac{1}{6}5*2.50(1.00+2*0.625)[2]+\frac{1}{3}5*2.50*0.625[1]=11.98$$

$$7.37X_1+11.98=0\rightarrow X_1=-1.625$$

$$\bar{M}=\bar{M}_0-1.625M_1$$



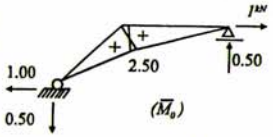
$$EI_c\delta_x=\int M\bar{M}\left[\frac{I_c}{I}\right]ds=\frac{1}{6}5(2*118.6*1.625-118.6*1.485-94.7*1.625+2*94.7*1.485)[2]$$

$$\ldots\ldots\ldots\ldots+\frac{1}{3}5*94.7*1.485*[1]+\frac{1}{3}5*62.5*1.485*[1]$$

$$EI_c\delta_x=950.0$$

$$EI_c=2*100.000=200.000kNm^2\rightarrow\delta_x=\frac{950.0}{200.000}=0.0048m=0.48cm$$

b2) Kısaltma teoreminin uygulanması ve $(\bar{M})_0$ diyagramının izostatik esas sistemden alınması hali



$$EI_c\delta_x=\int M(\bar{M})_0\left[\frac{I_c}{I}\right]ds=\frac{1}{6}5*2.5*(-118.6+2*94.7)[2]$$

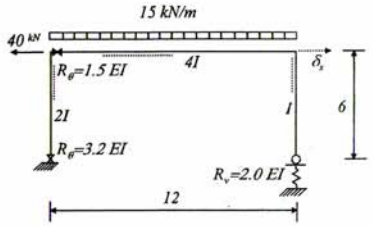
$$+\frac{1}{3}5*2.50*94.7[1]+\frac{1}{3}5*2.50*62.5[1]$$

$$EI_c\delta_x=950.0\rightarrow\delta_x=0.0048m$$

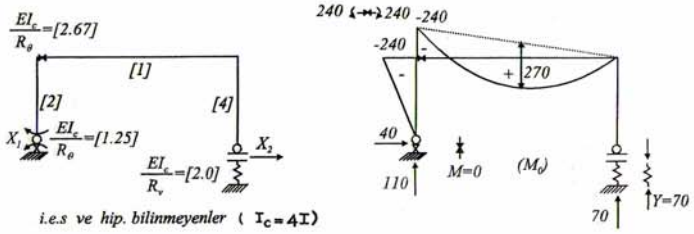
ÖRNEK: Şekildeki elastik mesnet ve elastik birleşimli sistemin

a) M diyagramını çiziniz

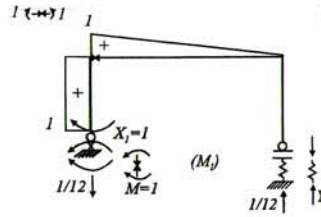
b) δ yatay yerdeğiştirmesini EI cinsinden hesaplayınız.

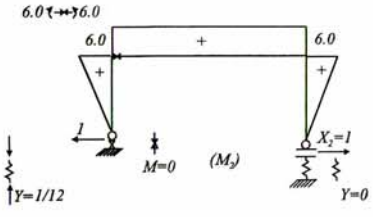


Çözüm:



i.e.s ve hip. bilinmeyenler ($I_c = 4I$)





$$EI_c\delta_{11} = \int M_1^2\left[\frac{I_c}{I}\right]ds + \sum M_1^2\frac{EI_c}{R_\theta} + \sum Y_1^2\frac{EI_c}{R_v} = 6*1^2[2] + \frac{1}{3}12*1^2[1] +$$

$$\underbrace{1^2[1.25] + 1^2[2.67]}_{\sum M_1^2\frac{EI_c}{R_\theta}} + \underbrace{\left(\frac{1}{12}\right)^2[2]}_{\sum Y_1^2\frac{EI_c}{R_v}} = 16.00 + 3.93 = 19.93$$

$$EI_c\delta_{12} = \frac{1}{2}6*1*6[2] + \frac{1}{2}12*1*6[1] + 0 + 1*6[2.67] + 0 = 72.00 + 16.00 = 88.00$$

$$EI_c\delta_{22} = \frac{1}{3}6*6^2[2] + 12*6^2[1] + \frac{1}{3}6*6^2[4] + 0 + 6^2[2.67] + 0 = 864.00 + 96.00 = 960.00$$

$$EI_c\delta_{10} = \int M_1M_0\left[\frac{I_c}{I}\right]ds + \sum M_1M_0\frac{EI_c}{R_\theta} + \sum Y_1Y_0\frac{EI_c}{R_v} = -\frac{1}{2}6*1*240[2] - \frac{1}{3}12*1*240[1] +$$

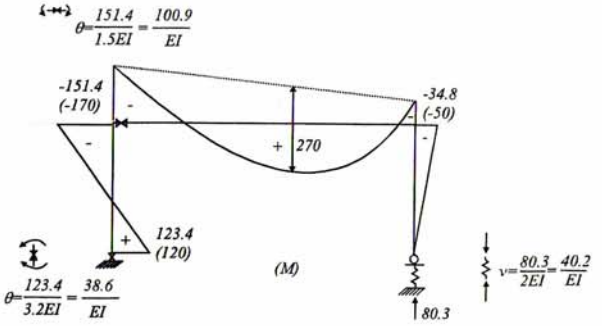
$$+\frac{1}{3}12*1*270[1] - 1*240[2.67] + \frac{1}{12}70*2 = -1320.0 - 628.3 = -1948.3$$

$$EI_c\delta_{20} = -\frac{1}{3}6*6*240[2] - \frac{1}{2}12*6*240[1] + \frac{2}{3}12*6*270[1] - 6*240*[2.67]$$

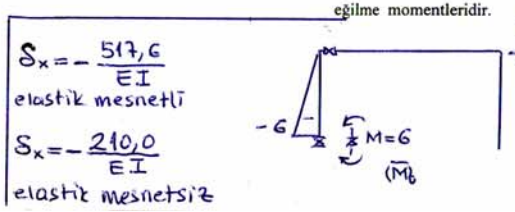
$$EI_c\delta_{20} = -1440.0 - 3840.0 = -5280.0$$

Denklem takımı;
$$\left.\begin{aligned} 19.93X_1 + 88.00X_2 - 1948.3 = 0 \\ 88.00X_1 + 960.00X_2 - 5280.0 = 0 \end{aligned}\right\} \rightarrow \begin{aligned} X_1 &= 123.4 \\ X_2 &= -5.80 \end{aligned}$$

$M = M_0 + 123.4M_1 - 5.80M_2$



Not : Parantez içinde verilen değerler, elastik mesnet ve birleşim içermeyen sistemin eğilme momentleridir.



Kısaltma Teoremine Göre

$$EI_cS_x = \int M(\bar{M}_0)\left[\frac{I_c}{I}\right]ds + \Sigma M(\bar{M}_b)\left[\frac{EI_c}{R_\theta}\right] + \Sigma Y(\bar{Y})_0\left[\frac{EI_c}{R_v}\right]$$

$$EI_cS_x = -\frac{1}{6}\times 6\times 6\times(2\times 123.4 - 151.4)\times[2] - 6\times 123.4\times[1.25] = -2070.3$$